Contribution ID: **50** Type: **not specified**

The complexity of Presburger arithmetic with power or powers

Tuesday, July 11, 2023 4:20 PM (20 minutes)

Michael Benedikt, Dmitry Chistikov, and Alessio Mansutti

Abstract: We investigate expansions of Presburger arithmetic (PA), i.e., the theory of the integers with addition and order, with additional structure related to exponentiation: either a function that takes a number to the power of 2, or a predicate P for the powers of 2.
The latter theory, denoted as PA(Pow), was introduced by Buchi as a first attempt at characterising the sets of tuples of numbers that can be expressed using finite automata; Buchi's method does not give an elementary upper bound, and the complexity of this theory has been open. The former theory, denoted as PA(Exp), was shown decidable by Semenov; while the decision procedure for this theory differs radically from the automata-based method proposed by Buchi, the method is also non-elementary. And in fact, the theory with the power function has a non-elementary lower bound. In this paper, we show that while Semenov's and Buchi's approaches yield non-elementary blow-ups for PA(Pow), the theory is in fact decidable in triply exponential time, similar to the best known quantifier-elimination algorithm for PA.
We also provide a NEXPTIME upper bound for the existential fragment of PA(Exp), a step towards a finer-grained analysis of its complexity.
Both these results are established by analysing a single parameterized satisfiability algorithm for PA(Exp), which can be specialized to either the setting of PA(Pow) or the existential theory of PA(Exp).
Besides the new upper bounds for the existential theory of PA(Exp) and PA(Pow), we believe our algorithm provides new intuition for the decidability of these theories, and for the features that lead to non-elementary blow-ups.

Presenters: MANSUTTI, Alessio; CHISTIKOV, Dmitry

Session Classification: Track B