

# On the complexity of diameter and related problems in permutation groups

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Abstract: We prove that it is  $\Pi_2^P$ -complete to verify whether the diameter of a given permutation group  $G = \langle A \rangle$  is bounded by a unary encoded number  $k$ . This solves an open problem from a paper of Even and Goldreich, where the problem was shown to be NP-hard. Verifying whether the diameter is exactly  $k$  is complete for the class consisting of all intersections of a  $\Pi_2^P$ -language and a  $\Sigma_2^P$ -language. A similar result is shown for the length of a given permutation  $\pi$ , which is the minimal  $k$  such that  $\pi$  can be written as a product of at most  $k$  generators from  $A$ . Even and Goldreich proved that it is NP-complete to verify, whether the length of a given  $\pi$  is at most  $k$  (with  $k$  given in unary encoding). We show that it is DP-complete to verify whether the length is exactly  $k$ .

Finally, we deduce from our result on the diameter that it is  $\Pi_2^P$ -complete to check whether a given finite automaton with transitions labelled by permutations from  $S_n$  produces all permutations from  $S_n$ .

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