On the complexity of diameter and related problems in permutation groups

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Abstract: We prove that it is $\Pi_2^{\rm P}$ -complete to verify whether the diameter of a given permutation group $G = \langle A \rangle$ is bounded by a unary encoded number k. This solves an open problem from a paper of Even and Goldreich, where the problem was shown to be NP-hard. Verifying whether the diameter is exactly k is complete for the class consisting of all intersections of a $\Pi_2^{\rm P}$ -language and a $\Sigma_2^{\rm P}$ -language. A similar result is shown for the length of a given permutation π , which is the minimal k such that π can be written as a product of at most k generators from A. Even and Goldreich proved that it is NP-complete to verify, whether the length of a given π is at most k (with k given in unary encoding). We show that it is DP-complete to verify whether the length is exactly k.

Finally, we deduce from our result on the diameter that it is Π_2^P -complete to check whether a given finite automaton with transitions labelled by permutations from S_n produces all permutations from S_n .

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