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Improved Approximation Algorithms by Generalizing the Primal-Dual Method Beyond Uncrossable Functions

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Abstract: We address long-standing open questions raised by Williamson, Goemans, Vazirani and Mihail pertaining to the design of approximation algorithms for problems in network design via the primal-dual method (Combinatorica 15(3):435-454, 1995). Williamson, et al., prove an approximation guarantee of two for connectivity augmentation problems where the connectivity requirements can be specified by so-called uncrossable functions. They state: "Extending our algorithm to handle non-uncrossable functions remains a challenging open problem. The key feature of uncrossable functions is that there exists an optimal dual solution which is laminar. This property characterizes uncrossable functions \dots\ A larger open issue is to explore further the power of the primal-dual approach for obtaining approximation algorithms for other combinatorial optimization problems."

Our main result proves a 16-approximation guarantee via the primal-dual method for a class of functions that generalizes the notion of an uncrossable function. We mention that the support of every optimal dual solution could be non-laminar for instances that can be handled by our methods.

We present a few applications of our main result to problems in the area of network design.

(a) A 16-approximation algorithm for augmenting the family of near-minimum cuts of a graph G.
The previous best approximation guarantee was $O(\log |V(G)|)$.

(b) A 20-approximation algorithm for the model of $(p, 2)$-Flexible Graph Connectivity.
The previous best approximation guarantee was $O(\log |V(G)|)$, where G denotes the input graph.

(c) A $16 \cdot \lceil k/u_{min} \rceil$-approximation algorithm for the Cap-k-ECSS problem which is as follows:
Given an undirected graph $G = (V, E)$ with edge costs c and edge capacities u, find a minimum cost subset of the edges $F \subseteq E$ such that the capacity across any cut in (V, F) is at least k;
u_{min} (respectively, u_{max}) denote the minimum (respectively, maximum) capacity of an edge in E, and w.l.o.g.\ $u_{max} \leq k$.
The previous best approximation guarantee was $\min(O(\log n), k, 2u_{max})$.

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