

# The Geometry of Tree-Based Sorting

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**Abstract:** We study the connections between sorting and the binary search tree (BST) model, with an aim towards showing that the fields are connected more deeply than is currently appreciated. While any BST can be used to sort by inserting the keys one-by-one, this is a very limited relationship and importantly says nothing about parallel sorting. We show what we believe to be the first formal relationship between the BST model and sorting. Namely, we show that a large class of sorting algorithms, which includes mergesort, quicksort, insertion sort, and almost every instance-optimal sorting algorithm, are equivalent in cost to offline BST algorithms. The main tool we use to bridge the gap between these areas is the geometric interpretation of the BST model introduced by Demaine et al., which finds an equivalence between searches on a BST and point sets in the plane satisfying a certain property. To give an example of the utility of our approach, we introduce the log-interleave bound, a measure of the information-theoretic complexity of a permutation  $\pi$ , which is within a  $\lg \lg n$  multiplicative factor of a known lower bound in the BST model; we also devise a parallel sorting algorithm with polylogarithmic span that sorts a permutation  $\pi$  using comparisons proportional to its log-interleave bound. Our aforementioned result on sorting and offline BST algorithms can be used to show existence of an offline BST algorithm whose cost is within a constant factor of the log-interleave bound of any permutation  $\pi$ .

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