

Parameter estimation for Gibbs distributions

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Abstract: A central problem in computational statistics is to convert a procedure for \emph{sampling} combinatorial objects into a procedure for \emph{counting} those objects, and vice versa. We will consider sampling problems which come from \emph{Gibbs distributions}, which are families of probability distributions over a discrete space Ω with probability mass function of the form $\mu_\beta^\Omega(\omega) \propto e^{\beta H(\omega)}$ for β in an interval $[\beta_{min}, \beta_{max}]$ and $H(\omega) \in \{0\} \cup [1, n]$.

The \emph{partition function} is the normalization factor $Z(\beta) = \sum_{\omega \in \Omega} e^{\beta H(\omega)}$, and the \emph{log partition ratio} is defined as $q = \frac{\log Z(\beta_{max})}{Z(\beta_{min})}$

We develop a number of algorithms to estimate the counts c_x using roughly $\tilde{O}(\frac{q}{\epsilon^2})$ samples for general Gibbs distributions and $\tilde{O}(\frac{n^2}{\epsilon^2})$ samples for integer-valued distributions (ignoring some second-order terms and parameters). We show this is optimal up to logarithmic factors. We illustrate with improved algorithms for counting connected subgraphs and perfect matchings in a graph.

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