

# New PRGs for Unbounded-width/Adaptive-order Read-once Branching Programs

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Lijie Chen, Xin Lyu, Avishay Tal and Hongxun Wu

Abstract: We give the first pseudorandom generators with sub-linear seed length for the following variants of read-once branching programs (roBPs):

1. First, we show there is an explicit PRG of seed length  $O(\log^2(n/\epsilon) \log(n))$  that fools unbounded-width unordered permutation branching programs with a single accept state, where  $n$  is the length of the program. Previously, [Lee-Pyne-Vadhan RANDOM 2022] gave a PRG with seed length  $\Omega(n)$  for this class. For the ordered case, [Hoza-Pyne-Vadhan ITCS 2021] gave a PRG with seed length  $O((\log(n))\log(1/\epsilon))$ .
2. Second, we show there is an explicit PRG fooling unbounded-width unordered regular branching programs with a single accept state with seed length  $O(\sqrt{n \log(n/\epsilon)})$ . Previously, no non-trivial PRG (with seed length less than  $n$ ) was known for this class (even in the ordered setting). For the ordered case, [Bogdanov-Hoza-Prakriya-Pyne CCC 2022] gave an HSG with seed length  $O((\log(n))\log(1/\epsilon))$ .
3. Third, we show there is an explicit PRG fooling width  $w$  adaptive branching programs with seed length  $O(\log(n)\log^2(nw/\epsilon))$ . Here, the branching program can choose an input bit to read depending on its current state, while it is guaranteed that on any input  $x \in \{0,1\}^n$ , the branching program reads each input bit exactly once. Previously, no PRG with a non-trivial seed length is known for this class. We remark that there are some functions computable by constant-width adaptive branching programs but not by sub-exponential-width unordered branching programs.

In terms of techniques, we indeed show that the Forbes-Kelly PRG (with the right parameters) from [Forbes-Kelly FOCS 2018] already fools all variants of roBPs above. Our proof adds several new ideas to the original analysis of Forbes-Kelly, and we believe it further demonstrates the versatility of the Forbes-Kelly PRG.

**Presenter:** WU, Hongxun

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