

# The Communication Complexity of Set Intersection under Product Distributions

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**Abstract:** We consider a multiparty setting where  $k$  parties have private inputs  $X_1, \dots, X_k \subseteq [n]$  and wish to compute the intersection  $\bigcap_{\ell=1}^k X_\ell$  of their sets, using as little communication as possible. This task generalizes the well-known problem of set disjointness, where the parties are required only to determine whether the intersection is empty or not.

In the worst-case, it is known that the communication complexity of finding the intersection is the same as that of solving set disjointness, regardless of the size of the intersection: the cost of both problems is  $\Omega(n \log k + k)$  bits in the shared blackboard model, and  $\Omega(nk)$  bits in the coordinator model.

In this work we consider a realistic setting where the parties' inputs are independent of one another, that is, the input is drawn from a product distribution. We show that this makes finding the intersection significantly easier than in the worst-case: only  $\tilde{\Theta}((n^{1-1/k} (H(S) + 1)^{1/k}) + k)$  bits of communication are required, where  $H(S)$  is the Shannon entropy of the intersection  $S$ . We also show that the parties do not need to exactly know the underlying input distribution; if we are given in advance  $O(n^{1/k})$

samples from the underlying distribution  $\mu$ , we can learn enough about  $\mu$  to allow us to compute the intersection of an input drawn from  $\mu$  using expected communication  $\tilde{\Theta}((n^{1-1/k} E[|S|]^{1/k}) + k)$ , where  $|S|$  is the size of the intersection.

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**Session Classification:** Track A-4