

# Hardness of Finding Combinatorial Shortest Paths on Graph Associahedra

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**Abstract:** We prove that the computation of a combinatorial shortest path between two vertices of a graph associahedron, introduced by Carr and Devadoss, is NP-hard. This resolves an open problem raised by Cardinal. A graph associahedron is a generalization of the well-known associahedron. The associahedron is obtained as the graph associahedron of a path. It is a tantalizing and important open problem in theoretical computer science that the computation of a combinatorial shortest path between two vertices of the associahedron can be done in polynomial time, which is identical to the computation of the flip distance between two triangulations of a convex polygon, and the rotation distance between two rooted binary trees. Our result shows that a certain generalized approach to tackling this open problem is not promising. As a corollary of our theorem, we prove that the computation of a combinatorial shortest path between two vertices of a polymatroid base polytope cannot be done in polynomial time unless  $P = NP$ . Since a combinatorial shortest path on the matroid base polytope can be computed in polynomial time, our result reveals an unexpected contrast between matroids and polymatroids.

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