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Flipper games for monadically stable graph classes

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Abstract: A class of graphs C is monadically stable if for every unary expansion $\widehat{C}$ of C, one cannot encode - using first-order transductions - arbitrarily long linear orders in graphs from $\widehat{C}$. It is known that nowhere dense graph classes are monadically stable; these include classes of bounded maximum degree and classes that exclude a fixed topological minor. On the other hand, monadic stability is a property expressed in purely model-theoretic terms that is also suited for capturing structure in dense graphs.

In this work we provide a characterization of monadic stability in terms of the Flipper game: a game on a graph played by Flipper, who in each round can complement the edge relation between any pair of vertex subsets, and Connector, who in each round is forced to localize the game to a ball of bounded radius. This is an analog of the Splitter game, which characterizes nowhere dense classes of graphs (Grohe, Kreutzer, and Siebertz, J.~ACM~'17).

We give two different proofs of our main result. The first proof is based on tools borrowed from model theory, and it exposes an additional property of monadically stable graph classes that is close in spirit to definability of types. Also, as a byproduct, we show that monadic stability for graph classes coincides with monadic stability of existential formulas with two free variables, and we provide another combinatorial characterization of monadic stability via forbidden patterns. The second proof relies on the recently introduced notion of flip-wideness (Dreier, M\"ahlmann, Siebertz, and Toruńczyk, arXiv~2206.13765) and provides an efficient algorithm to compute Flipper's moves in a winning strategy.

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