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Canonical decompositions in monadically stable and bounded shrubdepth graph classes

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Abstract: We use model-theoretic tools originating from stability theory to derive a result we call the Finitary Substitute Lemma, which intuitively says the following. Suppose we work in a stable graph class $\mathcal{C}$, and using a first-order formula φ with parameters we are able to define, in every graph $G \in \mathcal{C}$, a relation R that satisfies some hereditary first-order assertion ψ. Then we are able to find a first-order formula φ' that has the same property, but additionally is {\em{finitary}}: there is finite bound $k \in \mathbb{N}$ such that in every graph $G \in \mathcal{C}$, different choices of parameters give only at most k different relations R that can be defined using φ'.

We use the Finitary Substitute Lemma to derive two corollaries about the existence of certain canonical decompositions in classes of well-structured graphs.

- We prove that in the Splitter game, which characterizes nowhere dense graph classes, and in the Flipper game, which characterizes monadically stable graph classes, there is a winning strategy for Splitter, respectively Flipper, that can be defined in first-order logic from the game history. Thus, the strategy is canonical.
- We show that for any fixed graph class $\mathcal{C}$ of bounded shrubdepth, there is an $O(n^2)$-time algorithm that given an n-vertex graph $G \in \mathcal{C}$, computes in an isomorphism-invariant way a structure H of bounded treedepth in which G can be interpreted. A corollary of this result is an $O(n^2)$-time isomorphism test and canonization algorithm for any fixed class of bounded~shrubdepth.

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