

Twin-width of Planar Graphs is at most 8, and at most 6 when Bipartite Planar

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Abstract: Twin-width is a structural width parameter introduced by Bonnet, Kim, Thomassé and Watrigant [FOCS 2020]. Very briefly, its essence is a gradual reduction (a contraction sequence) of the given graph down to a single vertex while maintaining limited difference of neighbourhoods of the vertices, and it can be seen as widely generalizing several other traditional structural parameters. Having such a sequence at hand allows us to solve many otherwise hard problems efficiently. Graph classes of bounded twin-width, in which appropriate contraction sequences are efficiently constructible, are thus of interest in combinatorics and in computer science. However, we currently do not know in general how to obtain a witnessing contraction sequence of low width efficiently, and published upper bounds on the twin-width in non-trivial cases are often “astronomically large”.

We focus on planar graphs, which are known to have bounded twin-width (already since the introduction of twin-width), but the first explicit “non-astronomical” upper bounds on the twin-width of planar graphs appeared just a year ago; namely the bound of at most 183 by Jacob and Pilipczuk [arXiv, January 2022], and 583 by Bonnet, Kwon and Wood [arXiv, February 2022]. A subsequent manuscript of Bekos et al. improved the bound down to 37 [arXiv, April 2022]. We prove that the twin-width of every planar graph is at most 8, and construct a witnessing contraction sequence in linear time. Note that the currently best lower-bound planar example is of twin-width 7, by Král and Lamaison [arXiv, September 2022]. We also prove that the twin-width of every bipartite planar graph is at most 6, and again construct a witnessing sequence in linear time.

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