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Faster parameterized algorithms for modification problems to minor-closed classes

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Abstract: Let $calG$ be a minor-closed graph class and let G be an n-vertex graph. We say that G is a {\em k-apex} of $calG$ if G contains a set S of at most k vertices such that $G \setminus S$ belongs to $calG$. Our first result is an algorithm that decides whether G is a k-apex of $calG$ in time $2^{\mathsf{poly}(k)} \cdot n^2$.
This algorithm improves the previous one, given by Sau, Stamoulis, and Thilikos [ICALP 2020, TALG 2022], whose running time was $2^{\mathsf{poly}(k)} \cdot n^3$. The {\em elimination distance} of G to $calG$, denoted by $\mathsf{ed}_{calG}(G)$, is the minimum number of rounds required to reduce each connected component of G to a graph in $calG$ by removing one vertex from each connected component in each round. Bulian and Dawar [Algorithmica 2017] proved the existence of an {\sf FPT}-algorithm, with parameter k, to decide whether $\mathsf{ed}_{calG}(G) \leq k$. This algorithm is based on the computability of the minor-obstructions and its dependence on k is not explicit.
We extend the techniques used in the first algorithm to decide whether $\mathsf{ed}_{calG}(G) \leq k$ in time $2^{2^{2^{\mathsf{poly}(k)}}} \cdot n^2$. This is the first algorithm for this problem with an explicit parametric dependence in k. In the special case where $calG$ excludes some apex-graph as a minor, we give two alternative algorithms, one running in time $2^{2^{calO(k^2 \log k)}} \cdot n^2$ and one running in time $2^{\mathsf{poly}(k)} \cdot n^3$.
As a stepping stone for these algorithms, we provide an algorithm that decides whether $\mathsf{ed}_{calG}(G) \leq k$ in time $2^{calO(\mathsf{tw} \cdot k + \mathsf{tw} \log \mathsf{tw})} \cdot n$, where tw is the treewidth of G. This algorithm combines the dynamic programming framework of Reidl, Rossmanith, Villaamil, and Sikdar [ICALP 2014] for the particular case where $calG$ contains only the empty graph (i.e., for treedepth) with the representative-based techniques introduced by Baste, Sau, and Thilikos [SODA 2020]. In all the algorithmic complexities above, {\sf poly} is a polynomial function whose degree depends on $calG$, while the hidden constants also depend on $calG$.
Finally, we provide explicit upper bounds on the size of the graphs in the minor-obstruction set of the class of graphs $calE_k(calG) = \{G \mid \mathsf{ed}_{calG}(G) \leq k\}$.

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