

Nearly Tight Spectral Sparsification of Directed Hypergraphs

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Abstract: Spectral hypergraph sparsification, an attempt to extend well-known spectral graph sparsification to hypergraphs, has been extensively studied over the past few years. For undirected hypergraphs, Kapralov, Krauthgamer, Tardos, and Yoshida~(2022) have proved an ε -spectral sparsifier of the optimal $O^*(n)$ size, where n is the number of vertices and O^* suppresses the ε^{-1} and $\log n$ factors. For directed hypergraphs, however, the optimal sparsifier size has not been known. Our main contribution is the first algorithm that constructs an $O^*(n^2)$ -size ε -spectral sparsifier for a weighted directed hypergraph. Our result is optimal up to the ε^{-1} and $\log n$ factors since there is a lower bound of $\Omega(n^2)$ even for directed graphs. We also show the first non-trivial lower bound of $\Omega(n^2/\varepsilon)$ for general directed hypergraphs. The basic idea of our algorithm is borrowed from the spanner-based sparsification for ordinary graphs by Koutis and Xu~(2016). Their iterative sampling approach is indeed useful for designing sparsification algorithms in various circumstances. To demonstrate this, we also present a similar iterative sampling algorithm for undirected hypergraphs that attains one of the best size bounds, enjoys parallel implementation, and can be transformed to be fault-tolerant.

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