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On Finding Constrained Independent Sets in Cycles

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Abstract: A subset of $[n] = \{1, 2, \ldots, n\}$ is called stable if it forms an independent set in the cycle on the vertex set $[n]$. In 1978, Schrijver proved via a topological argument that for all integers n and k with $n \geq 2k$, the family of stable k-subsets of $[n]$ cannot be covered by $n - 2k + 1$ intersecting families. We study two total search problems whose totality relies on this result.

In the first problem, denoted by Schrijver(n,k,m), we are given an access to a coloring of the stable k-subsets of $[n]$ with $m = m(n, k)$ colors, where $m \leq n - 2k + 1$, and the goal is to find a pair of disjoint subsets that are assigned to the same color. While for $m = n - 2k + 1$ the problem is known to be PPA-complete, we prove that for $m \leq \alpha \cdot n/k$, with α being any fixed constant, the problem admits an efficient algorithm. For $m = \lfloor n/2 \rfloor - 2k + 1$, we prove that the problem is efficiently reducible to the Kneser problem, for which no hardness result is known. Motivated by the relation between the problems, we investigate the family of unstable k-subsets of $[n]$, which might be of independent interest.

In the second problem, called Unfair Independent Set in Cycle, we are given ℓ subsets $V_1, \ldots, V_\ell$ of $[n]$, where $\ell \leq n - 2k + 1$ and $|V_i| \geq 2$ for all $i \in [\ell]$, and the goal is to find a stable k-subset S of $[n]$ satisfying the constraints $|S \cap V_i| \leq |V_i|/2$ for $i \in [\ell]$. We prove that the problem is PPA-complete and that its restriction to instances with $n = 3k$ is at least as hard as the Cycle plus Triangles problem, for which no efficient algorithm is known. On the contrary, we prove that there exists a constant c for which the restriction of the problem to instances with $n \geq c \cdot k$ can be solved in polynomial time.

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