

Optimal Decremental Connectivity in Non-Sparse Graphs

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Abstract: We present a dynamic algorithm for maintaining the connected and 2-edge-connected components in an undirected graph subject to edge deletions. The algorithm is Monte-Carlo randomized and processes any sequence of edge deletions in $O(m + n \text{poly}(\log n))$ total time. Interspersed with the deletions, it can answer queries to whether any two given vertices currently belong to the same (2-edge-)connected component in constant time. Our result is based on a general Monte-Carlo randomized reduction from decremental c -edge-connectivity to a variant of fully-dynamic c -edge-connectivity on a sparse graph.

For non-sparse graphs with $\Omega(n \text{poly}(\log n))$ edges, our connectivity and 2-edge-connectivity algorithms handle all deletions in optimal linear total time, using existing algorithms for the respective fully-dynamic problems. This improves upon an $O(m \log(n^2/m) + n \text{poly}(\log n))$ -time algorithm of Thorup [J.Alg. 1999], which runs in linear time only for graphs with $\Omega(n^2)$ edges.

Our constant amortized cost for edge deletions in decremental connectivity in non-sparse graphs should be contrasted with an $\Omega(\log n / \log \log n)$ worst-case time lower bound in the decremental setting [Alstrup, Husfeldt, and Rauhe FOCS'98] as well as an $\Omega(\log n)$ amortized time lower-bound in the fully-dynamic setting [Patrascu and Demaine STOC'04].

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