

Lasserre Hierarchy for Graph Isomorphism and Homomorphism Indistinguishability

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Abstract: We show that feasibility of the t -th level of the Lasserre semidefinite programming hierarchy for graph isomorphism can be expressed as a homomorphism indistinguishability relation. In other words, we define a class L_t of graphs such that graphs G and H are not distinguished by the t -th level of the Lasserre hierarchy if and only if they admit the same number of homomorphisms from any graph in L_t . By analysing the treewidth of graphs in L_t we prove that the $3t$ -th level of Sherali–Adams linear programming hierarchy is as strong as the t -th level of Lasserre. Moreover, we show that this is best possible in the sense that $3t$ cannot be lowered to $3t-1$ for any t . The same result holds for the Lasserre hierarchy with non-negativity constraints, which we similarly characterise in terms of homomorphism indistinguishability over a family L_t^+ of graphs. Additionally, we give characterisations of level- t Lasserre with non-negativity constraints in terms of logical equivalence and via a graph colouring algorithm akin to the Weisfeiler–Leman algorithm. This provides a polynomial time algorithm for determining if two given graphs are distinguished by the t -th level of the Lasserre hierarchy with non-negativity constraints.

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