

Ellipsoid Fitting Up to a Constant

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Abstract: Saunderson, Parrilo and Willsky asked the following elegant geometric question: what is the largest $m = m(d)$ such that there is an ellipsoid in \mathbb{R}^d that passes through v_1, v_2, \dots, v_m with high probability when v_i are chosen independently from the standard Gaussian distribution $N(0, Id)$. The existence of such an ellipsoid is equivalent to the existence of a positive semidefinite matrix X such that $v_i^T X v_i = 1$ for every $1 \leq i \leq m$ — a natural example of a random semidefinite program. SPW conjectured that $m = (1 - o(1))d^{2/4}$ with high probability. Very recently, Potechin, Turner, Venkat and Wein [10] and Kane and Diakonikolas [8] proved that $m \leq d^{2/4} / \log^{O(1)}(d)$ via a certain natural, explicit construction.

In this work, we give a substantially tighter analysis of their construction to prove that $m \leq d^{2/4}/C$ for an absolute constant $C > 0$. This resolves one direction of the SPW conjecture up to a constant. Our analysis proceeds via the method of Graphical Matrix Decomposition that has recently been used to analyze correlated random matrices arising in various areas. Our key new technical tool is a refined method to prove singular value upper bounds on certain correlated random matrices that are tight up to absolute dimension-independent constants. In contrast, all previous methods that analyze such matrices lose logarithmic factors in the dimension.

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