An Optimal Separation between Two Property Testing Models for Bounded Degree Directed Graphs

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Pan Peng and Yuyang Wang

Abstract: We revisit the relation between two fundamental property testing models for bounded-degree \emph{directed} graphs: the \emph{bidirectional} model in which the algorithms are allowed to query both the outgoing edges and incoming edges of a vertex, and the \emph{unidirectional} model in which only queries to the outgoing edges are allowed. Czumaj, Peng and Sohler [STOC 2016] showed that for directed graphs with both maximum indegree and maximum outdegree upper bounded by d, any property that can be tested with query complexity $O_{\varepsilon,d}(1)$ in the bidirectional model can be tested with $n^{1-\Omega_{\varepsilon,d}(1)}$ queries in the unidirectional model. In particular, if the proximity parameter ε approaches 0, then the query complexity of the transformed tester in the unidirectional model approaches n. It was left open if this transformation can be further improved or there exists any property that exhibits such an extreme separation.

We prove that testing \emph{subgraph-freeness} in which the subgraph contains k source components, requires $\Omega(n^{1-\frac{1}{k}})$ queries in the unidirectional model. This directly gives the first explicit properties that exhibit an $O_{\varepsilon,d}(1)$ vs $\Omega(n^{1-f(\varepsilon,d)})$ separation of the query complexities between the bidirectional model and unidirectional model, where $f(\varepsilon, d)$ is a function that approaches 0 as ε approaches 0. Furthermore, our lower bound also resolves a conjecture by Hellweg and Sohler [ESA 2012] on the query complexity of testing k-star-freeness.

Presenter: PENG, Pan

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