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Faster submodular maximization for several classes of matroids

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Monika Henzinger, Paul Liu, Jan Vondrák and Da Wei Zheng

Abstract: The maximization of submodular functions have found widespread application in areas such as machine learning, combinatorial optimization, and economics, where practitioners often wish to enforce various constraints; the matroid constraint has been investigated extensively due to its algorithmic properties and expressive power. Though tight approximation algorithms for general matroid constraints exist in theory, the running times of such algorithms typically scale quadratically, and are not practical for truly large scale settings. Recent progress has focused on fast algorithms for important classes of matroids given in explicit form. Currently, nearly-linear time algorithms only exist for graphic and partition matroids [EN19]. In this work, we develop algorithms for submodular maximization constrained by graphic, transversal matroids on dense graphs, or laminar matroids in time near-linear in the size of their representation. Our algorithms achieve an optimal approximation of $1 - 1/e - \epsilon$ and both generalize and accelerate the results of Ene and Nguyen [EN19]. In fact, the running time of our algorithm cannot be improved without improving the classic continuous greedy of Badanidiyuru and Vondrák [BV14].

To achieve near-linear running time, we make use of dynamic data structures that maintain bases with approximate maximum cardinality and weight under certain element updates. These data structures need to support a weight decrease operation and a novel FREEZE operation that allows the algorithm to freeze elements (i.e. force to be contained) in its basis regardless of future data structure operations. For the laminar matroid, we present a new dynamic data structure using the top tree interface of Alstrup et al. [AHLT05] that maintains the maximum weight basis under insertions and deletions of elements in $O(\log n)$ time. This data structure needs to support certain subtree query and path update operations that are performed every insertion and deletion that are non-trivial to handle in conjunction. For the transversal matroid the FREEZE operation corresponds to requiring the data structure to keep a certain set S of vertices matched, a property that we call S-stability. While there is a large body of work on dynamic matching algorithms, none are S-stable and maintain an approximate maximum weight matching under vertex updates. We give the first such algorithm for bipartite graphs with total running time linear (up to log factors) in the number of edges.

Presenter: ZHENG, Da Wei

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