Contribution ID: **124** Type: **not specified**

Zero-Rate Thresholds and New Capacity Bounds for List-Decoding and List-Recovery

Friday, July 14, 2023 10:30 AM (20 minutes)

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Abstract: In this work we consider the list-decodability and list-recoverability of arbitrary q-ary codes, for all integer values of $q \geq 2$. A code is called $(p, L)_q$-list-decodable if every radius pn Hamming ball contains less than L codewords; $(p, \ell, L)_q$-list-recoverability is a generalization where we place radius pn Hamming balls on every point of a combinatorial rectangle with side length ℓ and again stipulate that there be less than L codewords.

Our main contribution is to precisely calculate the maximum value of p for which there exist infinite families of positive rate $(p, \ell, L)_q$-list-recoverable codes, the quantity we call the \emph{zero-rate threshold}. Denoting this value by p_*, we in fact show that codes correcting a $p_* + \varepsilon$ fraction of errors must have size $O_\varepsilon(1)$, i.e., independent of n. Such a result is typically referred to as a "Plotkin bound." To complement this, a standard random code with expurgation construction shows that there exist positive rate codes correcting a $p_* - \varepsilon$ fraction of errors. We also follow a classical proof template (typically attributed to Elias and Bassalygo) to derive from the zero-rate threshold other tradeoffs between rate and decoding radius for list-decoding and list-recovery.

Technically, proving the Plotkin bound boils down to demonstrating the Schur convexity of a certain function defined on the q-simplex as well as the convexity of a univariate function derived from it. We remark that an earlier argument claimed similar results for q-ary list-decoding; however, we point out that this earlier proof is flawed.

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Session Classification: Track A-3