

Approximating Long Cycle Above Dirac's Guarantee

Tuesday, July 11, 2023 10:30 AM (20 minutes)

Fedor Fomin, Petr Golovach, Danil Sagunov and Kirill Simonov

Abstract: Parameterization above (or below) a guarantee is a successful concept in parameterized algorithms. The idea is that many computational problems admit “natural” guarantees bringing to algorithmic questions whether a better solution (above the guarantee) could be obtained efficiently. For example, for every boolean CNF formula on m clauses, there is an assignment that satisfies at least $m/2$ clauses. How difficult is it to decide whether there is an assignment satisfying more than $m/2 + k$ clauses? Or, if an n -vertex graph has a perfect matching, then its vertex cover is at least $n/2$. Is there a vertex cover of size at least $n/2 + k$ for some $k \geq 1$ and how difficult is it to find such a vertex cover? The above guarantee paradigm has led to several exciting discoveries in the areas of parameterized algorithms and kernelization. We argue that this paradigm could bring forth fresh perspectives on well-studied problems in approximation algorithms. Our example is the longest cycle problem. One of the oldest results in extremal combinatorics is the celebrated Dirac's theorem from 1952. Dirac's theorem provides the following guarantee on the length of the longest cycle: for every 2-connected n -vertex graph G with minimum degree $\delta(G) \leq n/2$, the length of a longest cycle L is at least $2\delta(G)$. Thus the “essential” part in finding the longest cycle is in approximating the “offset” $k = L - 2\delta(G)$. The main result of this paper is the above-guarantee approximation theorem for k . Informally, the theorem says that approximating the offset k is not harder than approximating the total length L of a cycle. In other words, for any (reasonably well-behaved) function f , a polynomial time algorithm constructing a cycle of length $f(L)$ in an undirected graph with a cycle of length L , yields a polynomial time algorithm constructing a cycle of length $2\delta(G) + \Omega(f(k))$.

Presenter: SIMONOV, Kirill

Session Classification: Track A-1