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Nearly-Linear Time LP Solvers and Rounding Algorithms for Scheduling Problems

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Abstract: We study nearly-linear time approximation algorithms for non-preemptive scheduling problems in two settings: the unrelated machine setting, and the identical machine with job precedence constraints setting, under the well-studied objectives such as makespan and weighted completion time. For many problems, we develop nearly-linear time approximation algorithms with approximation ratios matching the current best ones achieved in polynomial time.

Our main technique is linear programming relaxation. For the unrelated machine setting, we formulate mixed packing and covering LP relaxations of nearly-linear size, and solve them approximately using the nearly-linear time solver of Young. For the makespan objective, we develop a rounding algorithm with $(2+\epsilon)$-approximation ratio. For the weighted completion time objective, we prove the LP is as strong as the rectangle LP used by Im and Li. This leads to a nearly-linear time $(1.45+\epsilon)$-approximation for the problem.

For problems in the identical machine with precedence constraints setting, the precedence constraints can not be formulated as packing or covering constraints. To achieve the nearly-linear running time, we define a polytope for the constraints, and leverage the multiplicative weight update (MWU) method with an oracle which always returns solutions in the polytope.

Along the way of designing the oracle, we encounter the single-commodity maximum flow problem over a directed acyclic graph $G=(V,E)$, where sources and sinks have limited supplies and demands, but edges have infinite capacities. We develop a $\frac{1}{1+\epsilon}$-approximation for the problem in time $O\left(\frac{|E|}{\epsilon}\log|V|\right)$, which may be of independent interest.

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