

Searching for Regularity in Bounded Functions

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Abstract: Given a bounded function f on \mathbb{F}_2^n , this work seeks a large affine subspace U such that f , when restricted to $calU$, has small nontrivial Fourier coefficients.

It is not hard to see that a random function is such that all its Fourier coefficients are extremely small already, so we focus on a natural class structured functions.

For the natural class of Fourier degree d functions $f : \mathbb{F}_2^n \rightarrow [-1, 1]$, we show that there exists an affine subspace of dimension at least $\tilde{\Omega}(n^{1/d} k^{-2})$, wherein all of f 's nontrivial Fourier coefficients become smaller than 2^{-k} .

To complement this result, we show the existence of degree d functions with coefficients larger than $2^{-d \log n}$ when restricted to any affine subspace of dimension larger than $\Omega(dn^{1/(d-1)})$. In addition, we give explicit examples of functions with analogous but weaker properties.

Along the way, we provide multiple characterizations of the Fourier coefficients of functions restricted to subspaces of \mathbb{F}_2^n that may be useful in other contexts. Finally, we highlight applications and connections of our results to parity kill number and affine dispersers.

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