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Searching for Regularity in Bounded Functions

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Abstract: Given a bounded function f on $\mathbb{F}_2^n$, this work seeks a large affine subspace $\mathcal{U}$ such that f, when restricted to
$calU$, has small nontrivial Fourier coefficients.
It is not hard to see that a random function is such that all its Fourier coefficients are extremely small already, so we focus on a natural class structured functions.

For the natural class of Fourier degree d functions $f : \mathbb{F}_2^n \to [-1, 1]$, we show that there exists an affine subspace of dimension at least $\tilde{\Omega}(n^{1/d!}k^{-2})$, wherein all of f's nontrivial Fourier coefficients become smaller than 2^{-k}.
To complement this result, we show the existence of degree d functions with coefficients larger than $2^{-d \log n}$ when restricted to any affine subspace of dimension larger than $\Omega(dn^{1/(d-1)})$. In addition, we give explicit examples of functions with analogous but weaker properties.

Along the way, we provide multiple characterizations of the Fourier coefficients of functions restricted to subspaces of $\mathbb{F}_2^n$ that may be useful in other contexts. Finally, we highlight applications and connections of our results to parity kill number and affine dispersers.

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