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A Hyperbolic Extension of Kadison-Singer Type Results

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Abstract: In 2013, Marcus, Spielman, and Srivastava resolved the famous Kadison-Singer conjecture. It states that for n independent random vectors $v_1, \cdots, v_n$ that have expected squared norm bounded by ϵ and are in the isotropic position in expectation, there is a positive probability that the determinant polynomial $\det(xI - \sum_{i=1}^{n} v_i v_i^\top)$ has roots bounded by $(1+\sqrt{\epsilon})^2$. An interpretation of the Kadison-Singer theorem is that we can always find a partition of the vectors $v_1, \cdots, v_n$ into two sets with a low discrepancy in terms of the spectral norm (in other words, rely on the determinant polynomial).

In this paper, we provide two results for a broader class of polynomials, the hyperbolic polynomials. Furthermore, our results are in two generalized settings:
* The first one shows that the Kadison-Singer result requires a weaker assumption
* The second one relaxes the Kadison-Singer result's distribution assumption to the Strongly Rayleigh distribution.
To the best of our knowledge, the previous results only support determinant polynomials [Anari and Oveis Gharan'14, Kyng, Luh and Song'20]. It is unclear whether they can be generalized to a broader class of polynomials. In addition, we also provide a sub-exponential time algorithm for constructing our results.

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