

#CSP Equality Corresponds to Quantum Isomorphism – A Holant Viewpoint

Tuesday, July 11, 2023 3:30 PM (20 minutes)

Jin-Yi Cai and Ben Young

Abstract: Recently, Mančinska and Roberson proved that two graphs G and G' are quantum isomorphic if and only if they admit the same number of homomorphisms from all planar graphs. We extend this result to planar #CSP with any pair of sets \mathcal{F} and \mathcal{F}' of real-valued, arbitrary-arity constraint functions. Graph homomorphism is the special case where each of \mathcal{F} and \mathcal{F}' contains a single symmetric 0-1-valued binary constraint function. Our treatment uses the framework of planar Holant problems. To prove that quantum isomorphic constraint function sets give the same value on any planar #CSP instance, we apply a novel form of holographic transformation of Valiant, using the quantum permutation matrix \mathcal{U} defining the quantum isomorphism. Due to the noncommutativity of \mathcal{U} 's entries, it turns out that this form of holographic transformation is only applicable to planar Holant. To prove the converse, we introduce the quantum automorphism group $\text{Qut}(\mathcal{F})$ of a set of constraint functions \mathcal{F} , and characterize the intertwiners of $\text{Qut}(\mathcal{F})$ as the signature matrices of planar $\text{Holant}(\mathcal{F} | \mathcal{EQ})$ quantum gadgets. Then we define a new notion of (projective) connectivity for constraint functions and reduce arity while preserving the quantum automorphism group. Finally, to address the challenges posed by generalizing from 0-1 valued to real-valued constraint functions, we adapt a technique of Lovász in the classical setting for isomorphisms of real-weighted graphs to the setting of quantum isomorphisms.

Presenter: YOUNG, Ben

Session Classification: Track A-2