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#CSP Equality Corresponds to Quantum Isomorphism – A Holant Viewpoint

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Abstract: Recently, Mančinska and Roberson proved that two graphs G and G' are quantum isomorphic if and only if they admit the same number of homomorphisms from all planar graphs. We extend this result to planar #CSP with any pair of sets $\mathcal{F}$ and $\mathcal{F}'$ of real-valued, arbitrary-arity constraint functions. Graph homomorphism is the special case where each of $\mathcal{F}$ and $\mathcal{F}'$ contains a single symmetric 0-1-valued binary constraint function. Our treatment uses the framework of planar Holant problems. To prove that quantum isomorphic constraint function sets give the same value on any planar #CSP instance, we apply a novel form of holographic transformation of Valiant, using the quantum permutation matrix $\mathcal{U}$ defining the quantum isomorphism. Due to the noncommutativity of $\mathcal{U}$'s entries, it turns out that this form of holographic transformation is only applicable to planar Holant. To prove the converse, we introduce the quantum automorphism group $\mathrm{Qut}(\mathcal{F})$ of a set of constraint functions $\mathcal{F}$, and characterize the intertwiners of $\mathrm{Qut}(\mathcal{F})$ as the signature matrices of planar Holant$(\mathcal{F} \mid \mathcal{EQ})$ quantum gadgets. Then we define a new notion of (projective) connectivity for constraint functions and reduce arity while preserving the quantum automorphism group. Finally, to address the challenges posed by generalizing from 0-1 valued to real-valued constraint functions, we adapt a technique of Lovász in the classical setting for isomorphisms of real-weighted graphs to the setting of quantum isomorphisms.

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