

Positivity Problems for Reversible Linear Recurrence Sequences

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Abstract: It is a longstanding open problem whether there is an algorithm to decide the Positivity Problem for linear recurrence sequences (LRS) over the integers, namely whether given such a sequence, all its terms are non-negative. Decidability is known for LRS of order 5 or less, i.e., for those sequences in which every new term depends linearly on the previous five (or fewer) terms. For *\emph{simple}* LRS (i.e., those whose characteristic polynomial has no repeated roots), decidability of Positivity is known up to order 9.

In this paper, we focus on the important subclass of reversible LRS, i.e., those integer LRS $\langle u_n \rangle_{n=0}^{\infty}$ whose bi-infinite completion $\langle u_n \rangle_{n=-\infty}^{\infty}$ also takes exclusively integer values; a typical example is the classical Fibonacci (bi-)sequence $\langle \dots, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, \dots \rangle$. Our main results are that Positivity is decidable for reversible LRS of order 11 or less, and for simple reversible LRS of order 17 or less.

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